Hilroy

80 Pages
27.6 cm x 21.2 cm

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EXERCISE BOOK
CAHIER D'EXERCICES

NAME/NOM COLE

SUBJECT/SUJET PHYS 114

ASSEMBLED IN CANADA WITH IMPORTED MATERIALS
ASSEMBLÉ AU CANADA AVEC DES MATIÈRES IMPORTÉES

12107

Week 2

2D Kinematics

- ex: at a car factory, a crash test dummy is used to test airbags. If the dummy's head, which is travelling at 20 m/s (70 km/hr), hits the airbag and comes to a stop after 0.05 s, what acceleration is the dummy's head experiencing?

SOLUTION:

$V_i = 20 m/s$

$V_f = 0 m/s$

$\Delta t = 0.05 s$

$\Delta \vec{x} = ?$

$\vec{a} = ?$

$V_f = V_i + at$

$a = \frac{\Delta V}{\Delta t}$

$\vec{a} = -400 m/s^2$

- CORE kinematic Equations:

- $d = V_0 t + \frac{1}{2} a t^2$
- $V_f^2 = V_0^2 + 2ad$
- $V_f = V_0 + at$

Forces

(Newton 1^{st} Law)

$F_{NET} = 0 \Rightarrow$ Equilibrium

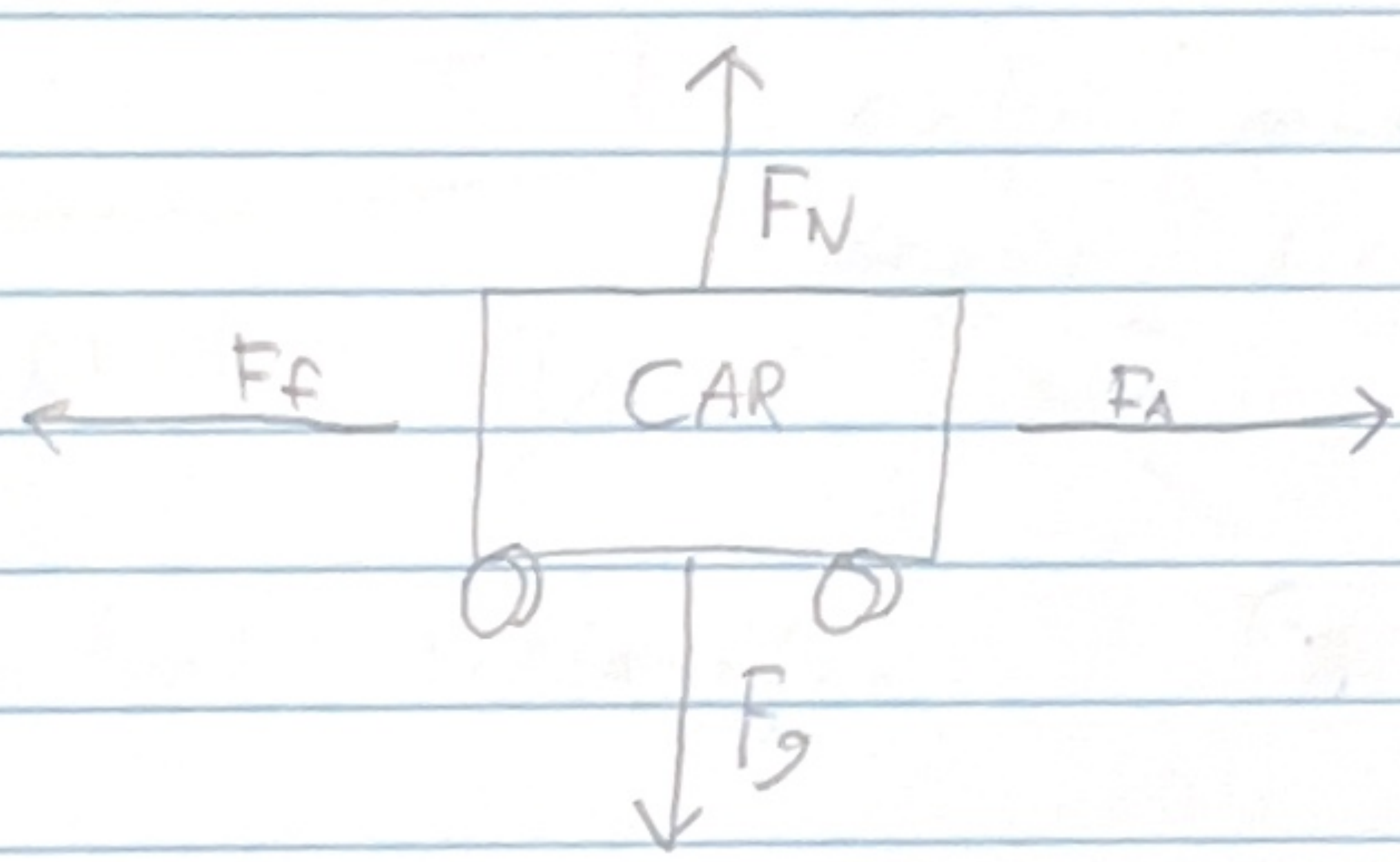
- Inertia / objects resist change in its motion

Static Equilibrium / Not moving

Dynamic Equilibrium / Moving but $F_{NET} = 0$

Normal Force (F_N) / perpendicular to contact surface

ex:

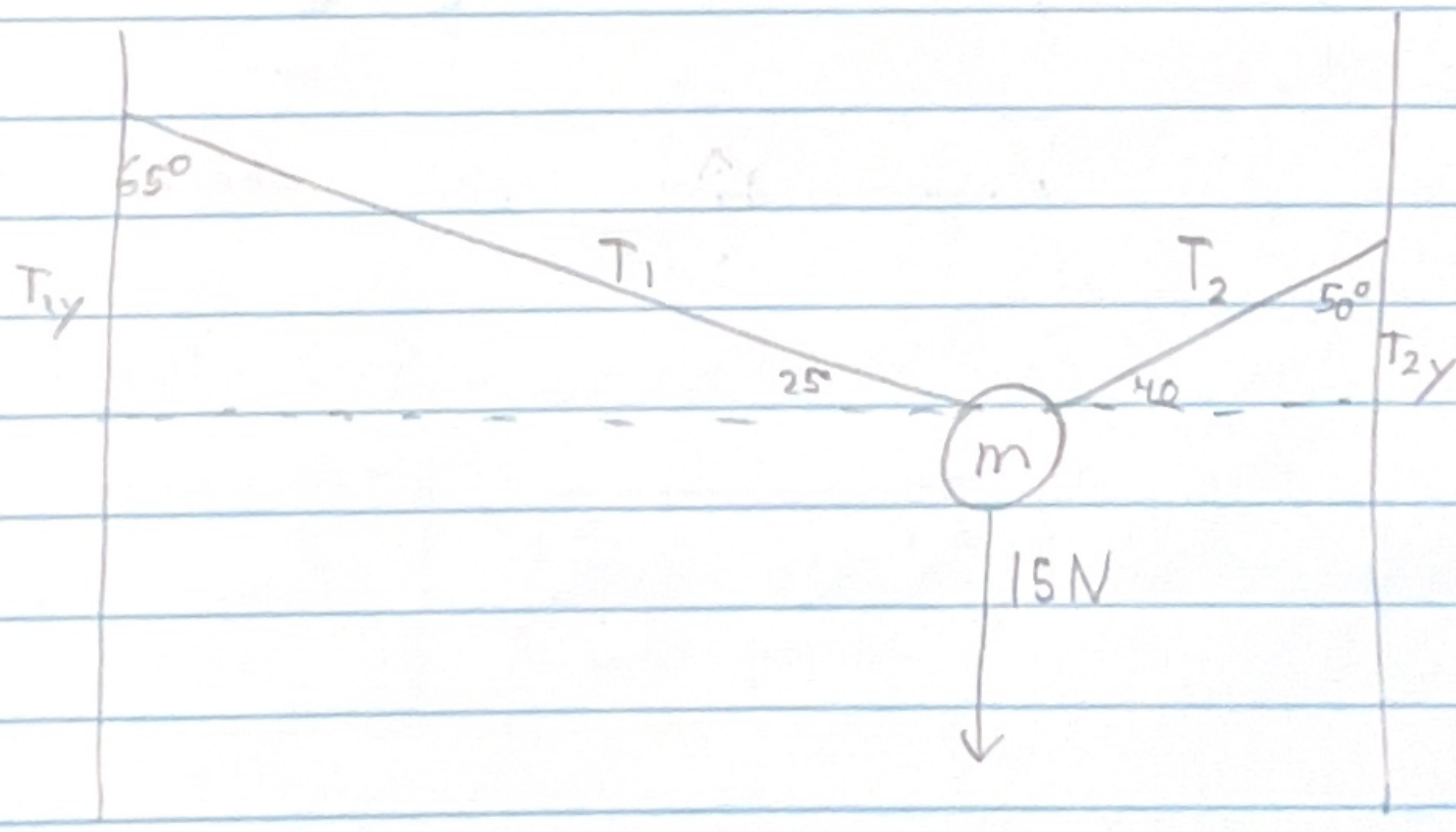


$F_{NET} = 0$

$T_1x - T_2x = 0$

$T_{1x} = -T_2x$

ex:



$T_{1y} + T_{2y} = -15N$

$T_{1y} = T_1 \sin 25$

$T_{1x} = T_1 \cos 25$

$T_{2y} = T_2 \sin 40$

$T_{2x} = T_2 \cos 40$

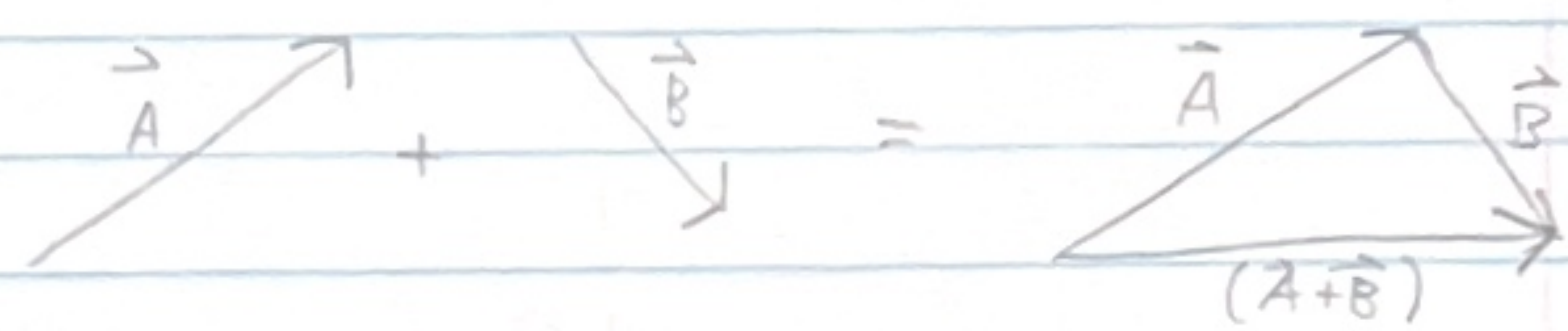
TEXT
Chp. 1

Concepts of Motion

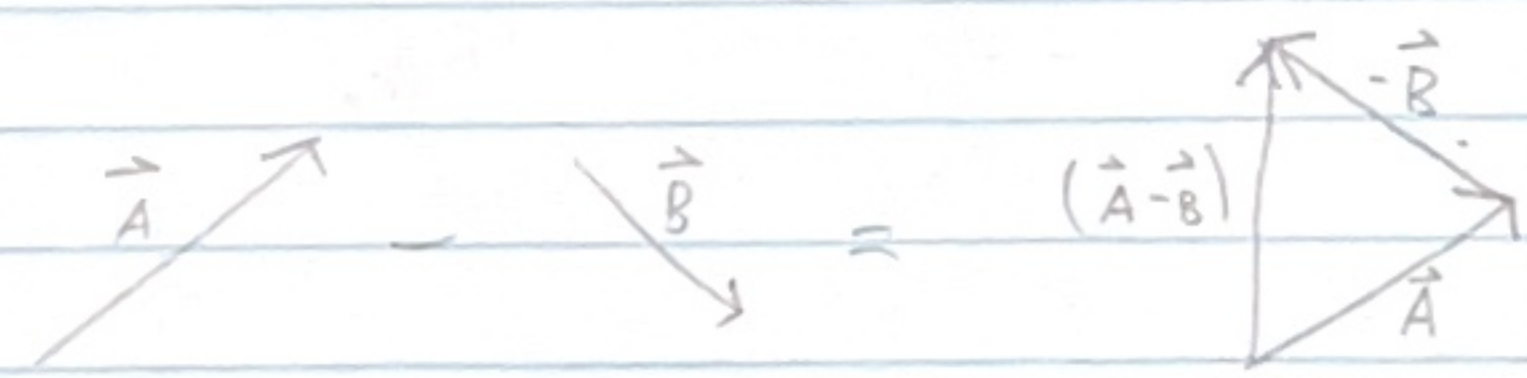
- 4 Types of Motion :/
  - Linear
  - Circular
  - Projectile
  - Rotational

- Vector Addition/Subtraction:

- Addition /



- Subtraction /



*(Invert A or B)

- Velocity:

$$\vec{V}_{avg} = \frac{\Delta d}{\Delta t}$$ where $\Delta d = d_f - d$

- Acceleration $\vec{a}_{(avg)} = \frac{\Delta \vec{V}}{\Delta t}$

- Circular Motion

- $\boxed{V = \frac{2\pi r}{T}}$ where V = velocity
  r = radius
  T = period (Time to go around once)

- Angular Velocity:

  $\boxed{|\omega| = \frac{2\pi}{T}}$, only magnitude. Need to know direction to get $\pm$.

- Tangential velocity:

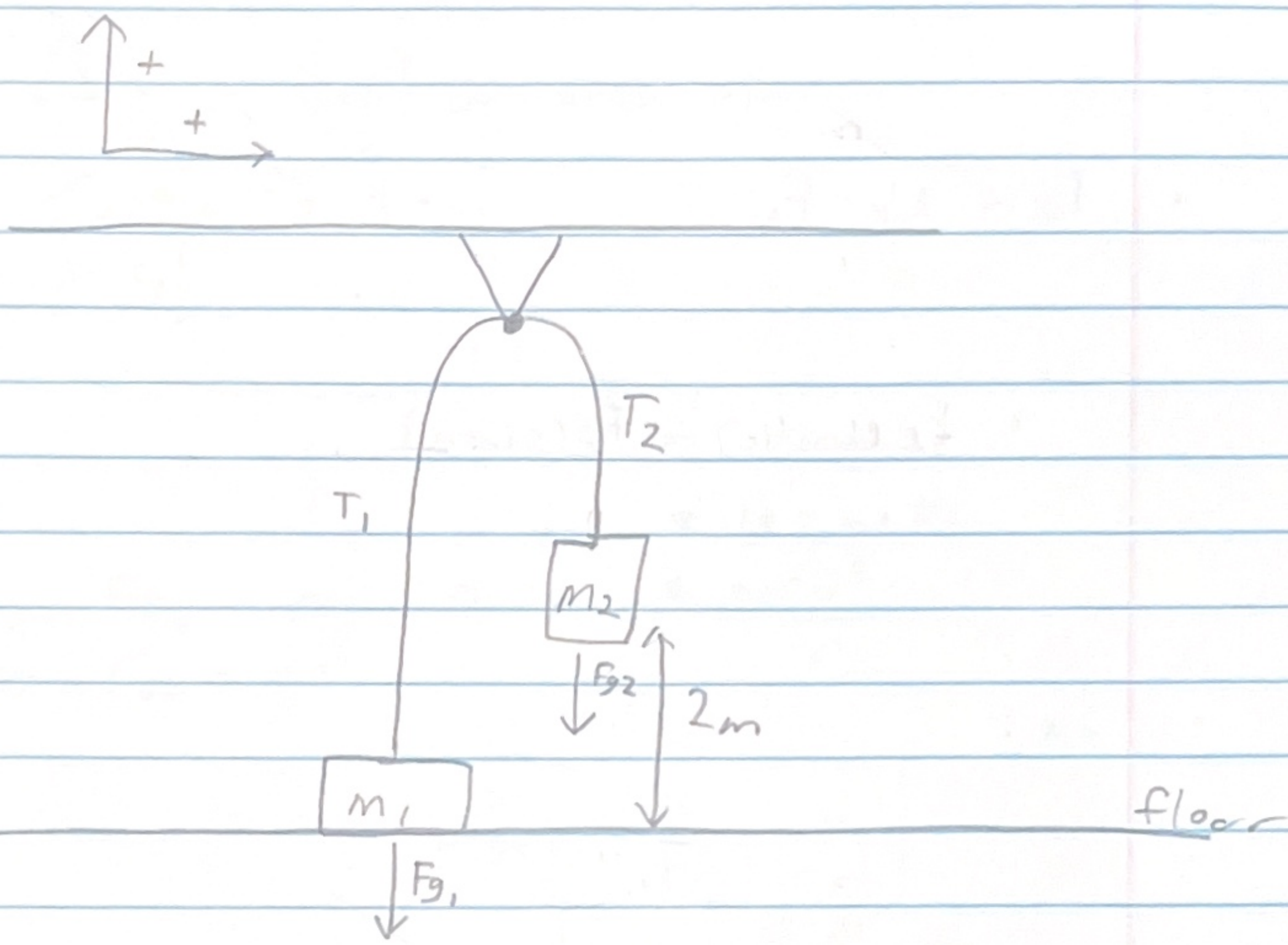
  $\boxed{V_t = \omega r}$

- Centripital acceleration:

  $\boxed{a_r = \frac{V^2}{r}}$ $\boxed{a_r = \omega^2 r}$ (* Towards centre *)
  (Radial)

  $a_{TOT} = \sqrt{a_r^2 + a_t^2}$

ex



$m_1 = 3kg$
$m_2 = 8kg$

$T_1 = T_2$
$\vec{a}_1 = -\vec{a}_2$

$\sum F_y = T_1 - mg_1 = m_1 a_1$

$\sum F_y = T - mg_2 = m_2 a_2$

$T - mg_2 = -m_2 a_1$

$(m_2 - m_1)g = (m_1 + m_2)a$

$a = \dfrac{(m_2 - m_1)g}{(m_1 + m_2)}$ $\therefore \vec{a} = 4.45 m/s^2$

For Pulley systems (For Now...)

$\vec{a}_x = \dfrac{\sum F_x}{(m_1 + m_2)}$

$\vec{a}_y = \dfrac{\sum F_x}{(m_1 + m_2)}$

Friction

(*coefficient of friction)

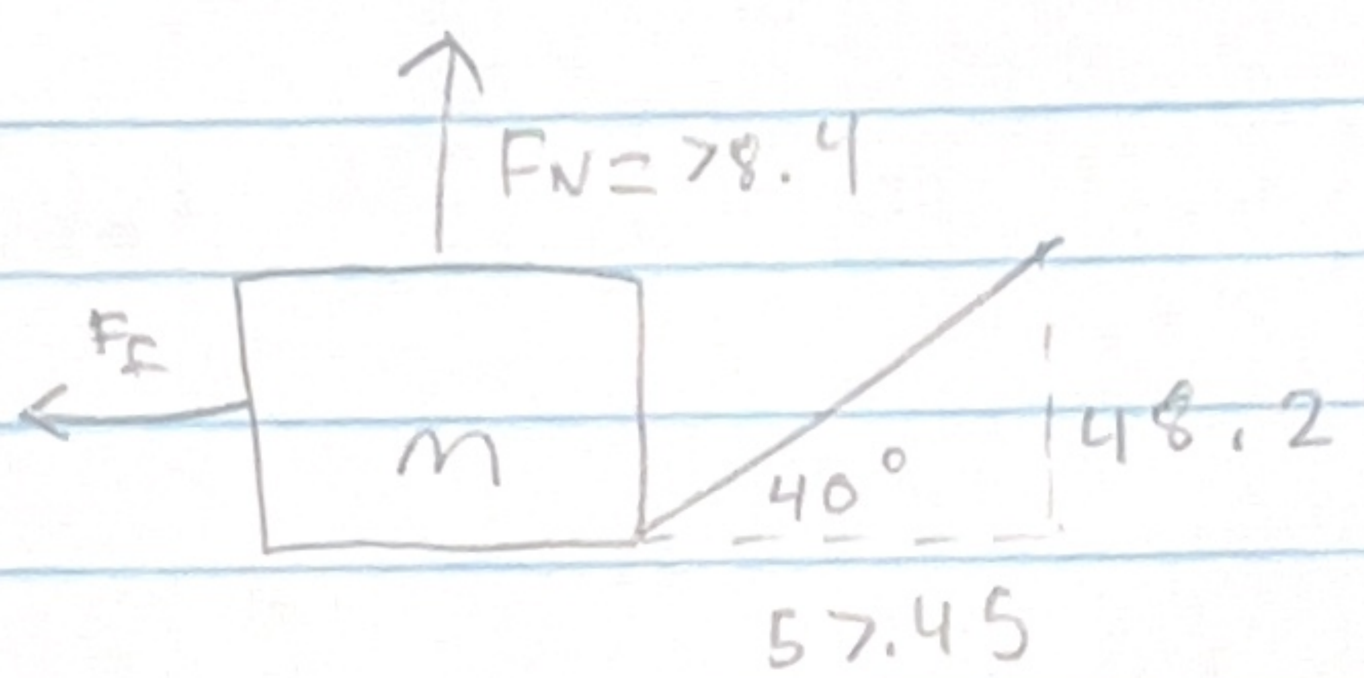
- $F_f = N_K \cdot F_N$

(* static coefficient)

- $F_s = \mu_s F_N$

- F_f (kinetic) < F_f (static)

- ex:

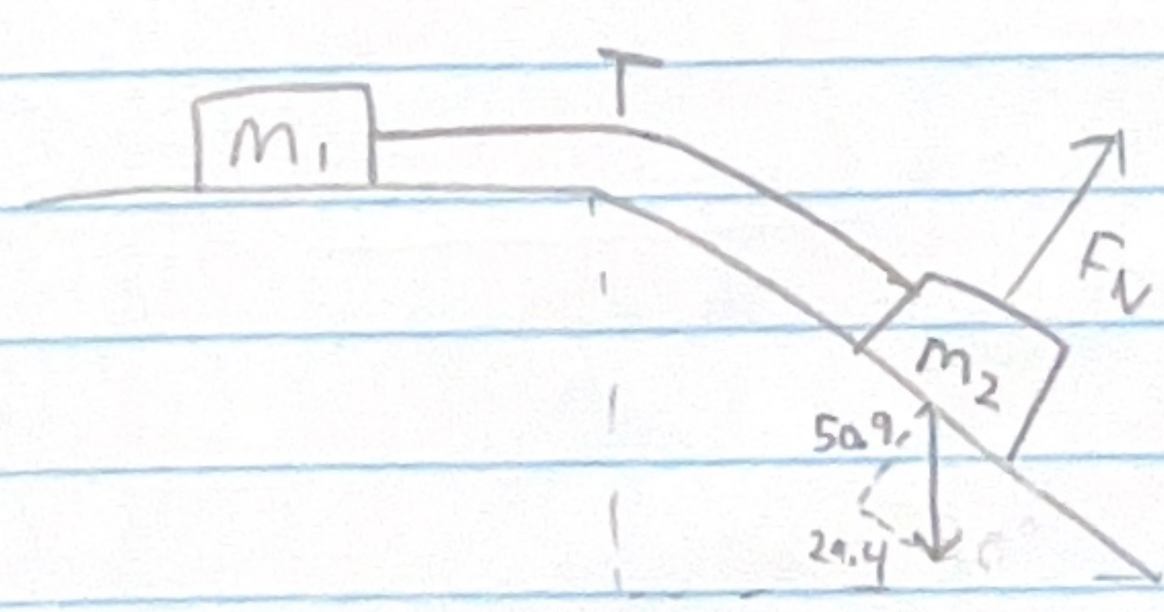


$F_f = (0.4)(78.4 - 57.45) = 12.076$

$F_A = 57.45 - 12.076 = 45.374$

$\therefore a = 5.67 \, m/s^2$ to the right

- ex:



$t_1 = t_2$

$\bar{a}_1 = \bar{a}_2$

$T = m_1 a(\mu k_1 g + 1)$

$F_{f(AL)} = (9.8)(2)(0.47) = 9.212 \, N$

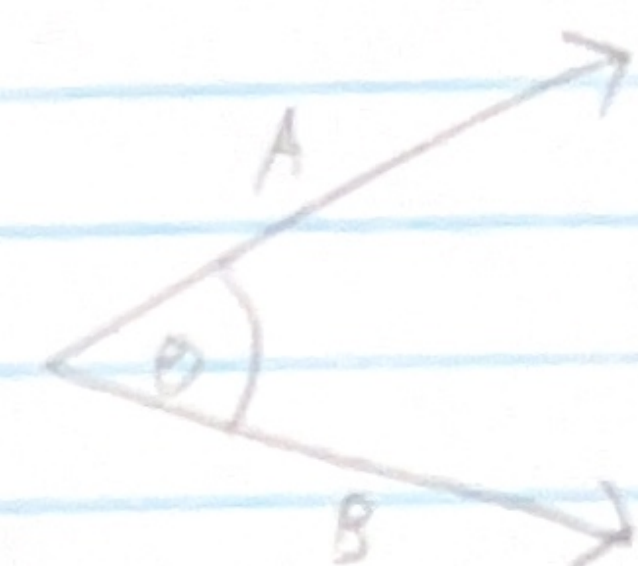
$F_{f(Cu)} = (50.9)(0.36) = 18.3$

$\Sigma F_x = 29.4 - 18.3 - 9.212 = \frac{1.888 \, N}{8 \, kg} = 0.236 \, m/s^2$ RIGH

LEC

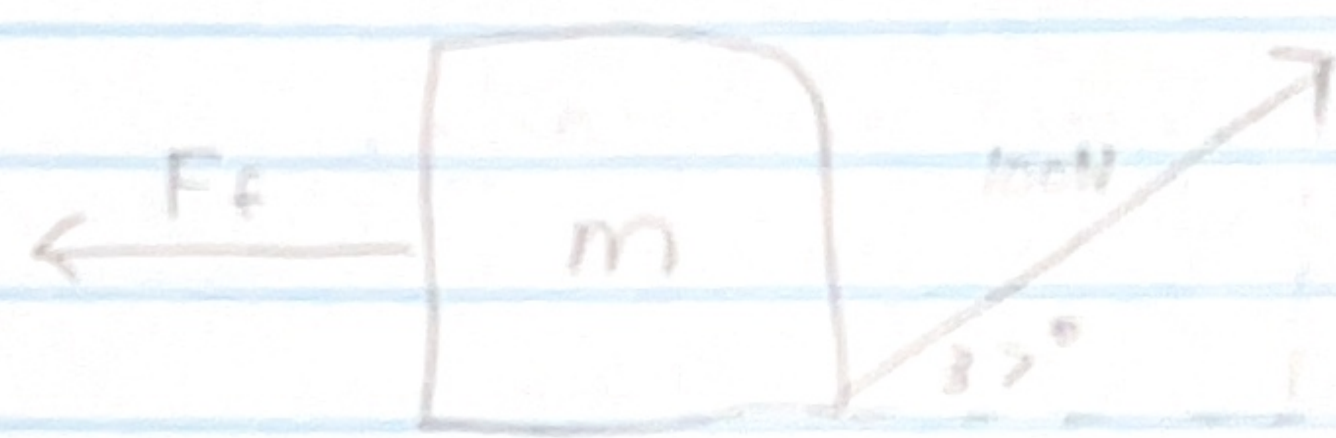
Work and Energy

- Vector Multiplication / $\bar{A}\cdot\bar{B} = AB\cos\theta$



- ex

m=50



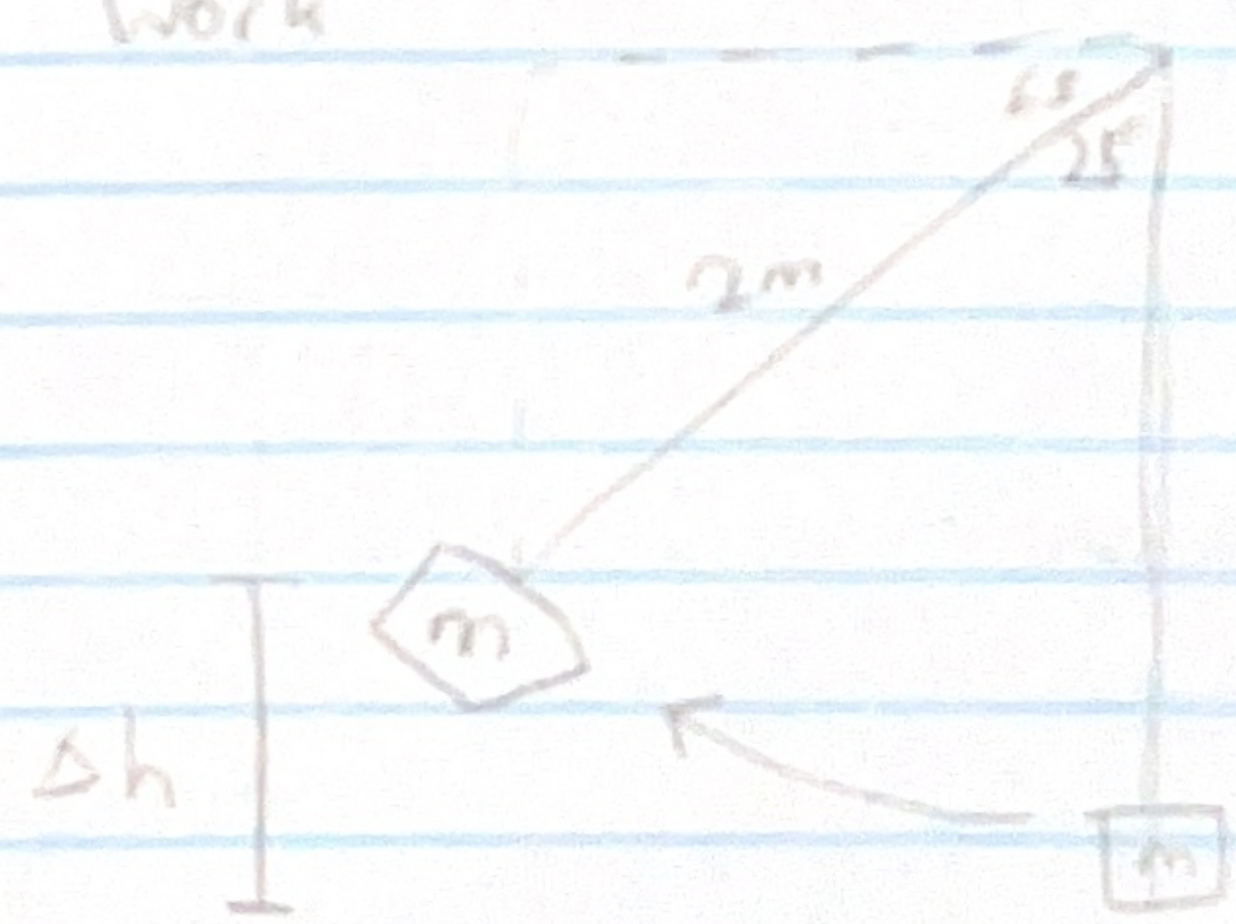
$$\Sigma F_x = 100\cos 37 - 50 = 26.5\,N$$

$$\text{Net Work} = \Delta x \cdot F = (40\,m)(26.5\,N) = 1061.66\ N\cdot m$$

Work

- ex

m=30



$2\sin 65 = 1.81\,m$

$1.81\,m - 2\,m = 0.1874\,m$

$\Delta PE = mgh = (30)(9.8)(0.1874) = \boxed{55.1\ N\cdot m}$

Conservation of Energy

Conservative Force / does not depend on path (W_c)
Non-Conservative Force / depends on path (W_{nc})

$\Sigma Work = \Delta E_k$

Total Work Energy Thereom

$$W_{nc} = (KE_f - KE_i) + (PE_f - PE_i)$$

$$E_{ki}\,E_{pi} = E_{kf}\,E_{pf} + \Delta h$$

ex:

$\Sigma F = -F_f$

$\Sigma F = -9.8$

$\Sigma F = ma$

$\frac{-9.8}{10} = a$

$\therefore a = -0.98 m/s^2$

$$E_{ki}\cancel{E_{pi}} = E_{kf}\cancel{E_{pf}} + \Delta h$$

$$E_{ki} = E_{kf} + \Delta h$$

$$\frac{1}{2}(10)(2^2) = 0 + \Delta h$$

$\therefore \Delta h = 20J$

$v_f = V_o + at$

$\therefore t = 2.04s$

$\Delta x = V_o t + \frac{1}{2}at^2$

$\Delta x = 2.04m$

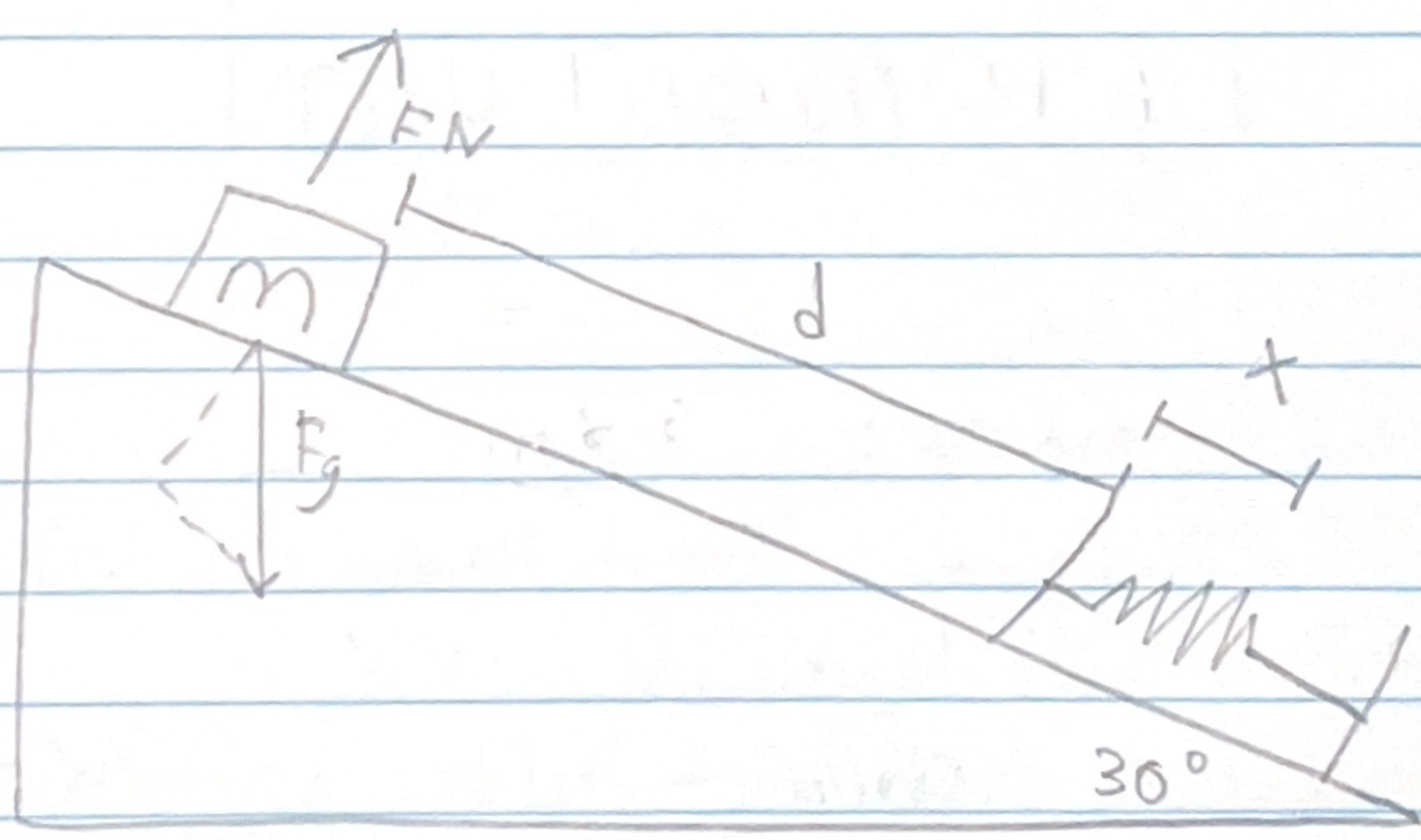
$$W_{nc} = \Delta PE + \Delta KE$$

$$= mgh + \frac{1}{2}kx_f^2 - \cancel{\frac{1}{2}kx_i^2} + \frac{1}{2}mv_f^2 - \cancel{\frac{1}{2}mv_i^2}$$

$$W_{nc} = -mgh - \frac{1}{2}kx_f^2 + \frac{1}{2}mv_f^2$$

$$F_f = -mgh - \frac{1}{2}kx_f^2 + \frac{1}{2}mv_f^2$$

$$-F_f \cdot \Delta x = -mgh - \frac{1}{2}kx_f^2 + \frac{1}{2}mv_f^2$$



a.) $\Sigma F_x = mg\sin 30 - mg\mu_k$

$\Sigma F_x = 44.1$

$\Sigma F_x = ma$

$\therefore a = 2.94\,m/s^2$

$v_f^2 = v_0^2 + 2a\Delta x$

$v_f = 5.42\,m/s$

b.) $W_{nc} = \Delta PE + \Delta KE$

$W_{nc} = \Delta PE_g + \Delta PE_s + \Delta KE$

$F_f \cdot d = mgh + \frac{1}{2}kx_f^2 - \cancel{\frac{1}{2}kx_i^2} + \frac{1}{2}mv_f^2 - \cancel{\frac{1}{2}mv_i^2}$

$F_f \cdot \Delta x = mgh + \frac{1}{2}kx_f^2 + \frac{1}{2}mv_f^2$

$(mg\sin 30)(\mu k)(6) = mgh + \frac{1}{2}kx_f^2 + \frac{1}{2}mv_f^2$

$\therefore k =$

$F_{net} = M\vec{a}_{cm}$

$\boxed{\text{Momentum}}$

Center of mass: $\frac{\Sigma m_i x_i}{m_{total}}$

y-direction $= \frac{m(0) + m(L\cos 30) + m(0)}{3m}$

$y_{cm} = \frac{L\cos 30}{3}$

$\bar{F}_{tot} = \sum_{i=1}^{N} \bar{F}_i = \sum_{i=1}^{N} \bar{F}_{i(ext)} + \cancel{\sum_{i=1}^{N} \bar{F}_{i(int)}}^{\,*=0}$

$F_{ext} = M \frac{d^2(\vec{r}_{cm})}{dt^2}$

LEC

Momentum

ex: $m_1 V_1 + m_2 V_2 = (m_1 + m_2) V_3$

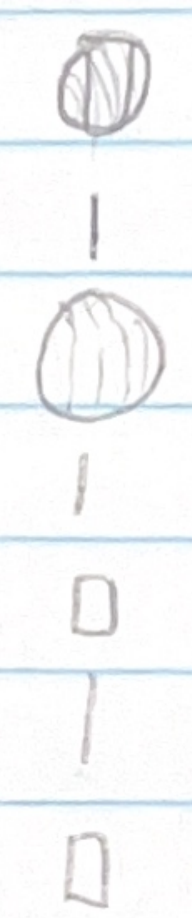
$m_1 = 80 kg$
$m_2 = 0.35 kg$
$V_1 = 0$
$V_2 = ?$
$V_3 = 4.9 m/s$

$\therefore V_2 = 1050 m/s$

$$\boxed{V_a' = \frac{(m_a - m_b) V_a}{m_a + m_b}}$$

$$\boxed{V_b' = \left(\frac{2 m_a}{m_a + m_b}\right) \cdot V_a}$$

ex:



$m_1 V_1 + m_2 V_2 = (m_1 + m_2) V_3$

$(0.03)(200) + 0 = (0.03 + 0.15) V_3$

$\therefore V_3 = 33.3 m/s$

$E_k = E_p$

$\frac{1}{2}(0.03 + \cancel{0}.15)(33.3)^2 = (0.03 + \cancel{0}.015)(9.81) h$

$\therefore h = 56.5 m$

LEC

Angular

(Average) • $\omega = \frac{\Delta\theta}{\Delta t}$

$\omega = \frac{V}{r}$

$V = \omega \cdot r$

(Instantaneous) • $\omega = \frac{d\theta}{dt}$

rad/s

TOTAL ACCELERATION

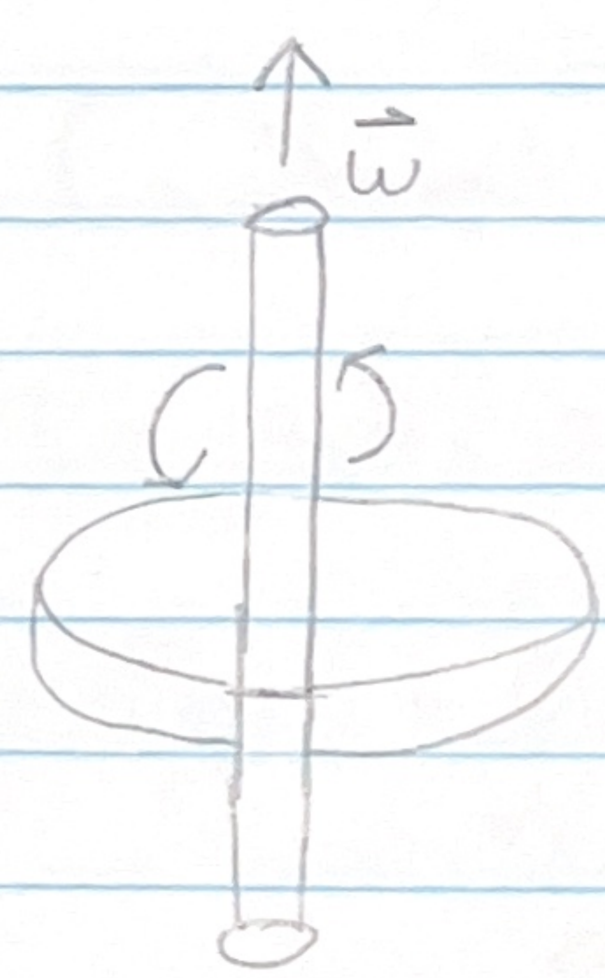
$a_{Total} = \sqrt{a_c^2 + a_t^2}$

$= \sqrt{r^2\omega^4 + r^2\alpha^2}$

Centripital acceleration

• $a_c = \frac{V_t^2}{r} = \omega^2 r$

• RIGHT HAND RULE (RHR)



Tangential Speed

• $\bar{V}_t^{avg} = r\bar{\omega}^{avg}$

• $V_t = r\omega$

Tangential acceleration

• $\bar{a}_t^{avg} = r\bar{\alpha}^{avg}$

• $a_t = r\alpha$

• Angular Acceleration

(Average) • $\vec{\alpha} = \frac{\Delta\omega}{\Delta t}$

(Instantaneous) $\vec{\alpha} = \frac{d\omega}{dt}$

Kinematics

$\Delta x = \theta, \ V = \omega, \ a = \alpha$

LEC

Torque

ex: A 15 cm metal rod laying along the x-axis is attatched to the origin at one end, A Force $F = (2.0\hat{i} + 1.5\hat{j} + 1.0\hat{k})$ N is applied to the rod at a point, 10cm away from its pivot point. Determine the torque on this object.

$\Delta r = 0.1 m\hat{i}$

$T = F \Delta r$

$$\begin{array}{ccc} \hat{i} & \hat{j} & \hat{k} \\ \hline 0.1 & 0 & 0 \\ 2 & 1.5 & 1 \end{array}$$

$= 0\hat{i} - (0.1)\hat{j} + 1.5\hat{k}$

$= (-0.1\hat{j} + 1.5\hat{k})$ N·m

$T = Fr\sin\theta$

Counter-clockwise (+) [out of page] $\odot$

Clockwise (-) [into the page] $\otimes$

$L_i = L_f$ * if no net torques (no external forces)

$L_{i_h} + L_{i_m} = L_{f_h} + L_{f_m}$

* (* Table)

$L = I \cdot \bar{\omega}$

$I = mR^2$

$m_h (r_i)^2 \omega_i + \frac{1}{2} M_m R_m^2 \omega_i = m_h (r_f)^2 \omega_f + \frac{1}{2} M_m R_m^2 \omega_f$

$(80)(2)^2(0.2) + \frac{1}{2}(25)(2)^2(0.2) = (80)(1)^2 \omega_f + \frac{1}{2}(25)(2)^2 \omega_f$

$\therefore \omega_f = 0.57$ rev/s

2

$L_i = L_f$

$L_{i\,Disk} + L_{i\,Rod} = L_{f\,Disk} + L_{f\,Rod}$

$I_{i\,Disk}\,\omega_i + I_{i\,Rod}\,\omega_i = I_{f\,Disk}\,\omega_f + I_{f\,Rod}\,\omega_f$

$L_i = \frac{1}{2} M_{Disk} \cdot R_{Disk}^2 \cdot \omega_i + \left(\frac{1}{2} m_{Rod}\, r^2 + m_{Rod}(R-r)^2\right)\omega_i$

$L_f = \frac{1}{2} M_{Disk} R_{Disk}^2 \cdot \omega_f + \frac{1}{12} m_{Rod} L^2 \omega_f$

$L_i = \frac{1}{2}(10)(25)^2(15) + \left(\frac{1}{2}(1.25)(2)^2 + (1.25)(25-2)^2\right)(15)$

$L_f = \frac{1}{2}(10)(25)^2 \boxed{\omega_f} + \frac{1}{12}(1.25)(2R)^2 \boxed{\omega_f}$

$\therefore \omega_f = 16.8$ rad/s

Rotational Kinetic Energy

$m = 4kg$

$R = 0.5m$

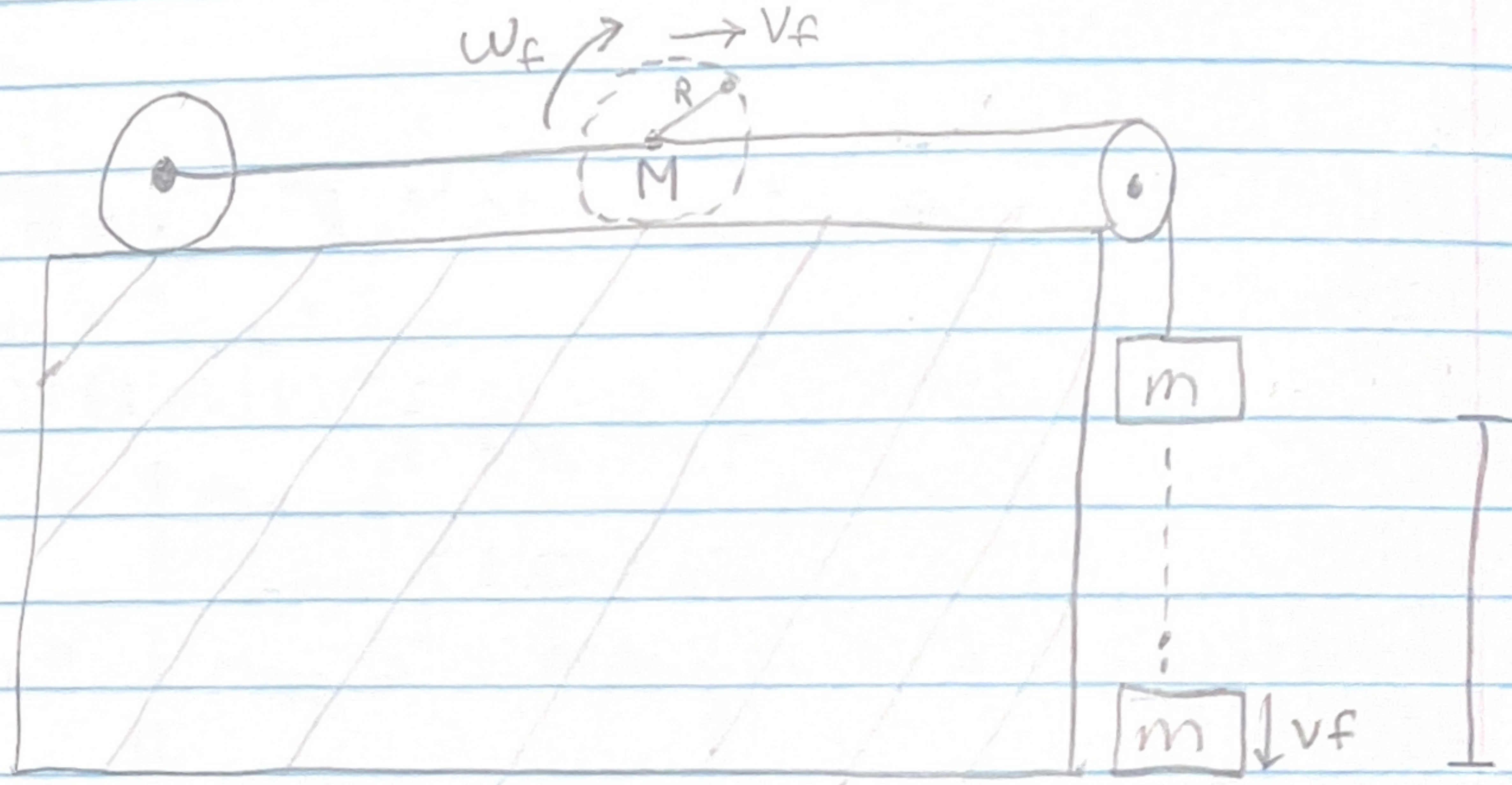
$M = 8kg$

Moment of Inertia $= 2 kg\cdot m^2$

$H = 2m$

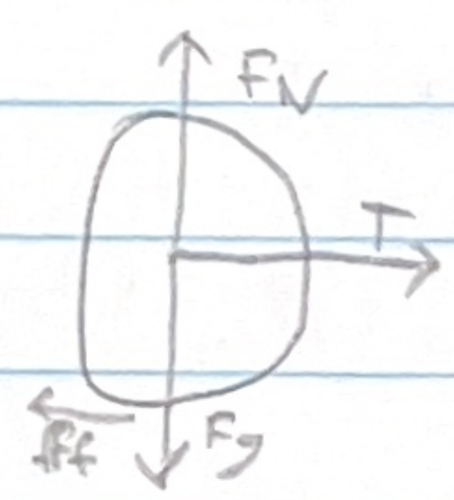
- $KE_r = \frac{1}{2} I \omega^2$

ex:



static friction is conservative

FBD:



$W_{nc} = \Delta Pe + \Delta KE_R$

$0 = mg\Delta y_0 + \frac{1}{2} m v_{f_B}^2 + \frac{1}{2} M v_f^2 + \frac{1}{2} I \omega_f^2$

$-\left(\frac{1}{2} m v_{f_B}^2 + \frac{1}{2} M v_f^2\right) = mg\Delta y_0 + \frac{1}{2} I \left(\frac{V}{R}\right)^2$

$v_f^2 \left(-\frac{1}{2} m + \frac{1}{2} M\right) = mg\Delta y_0 + \frac{1}{2} I \left(\frac{V}{R}\right.$

$\boxed{V_f = -2.8 \, m/s}$

LEC

Simple Harmonic Motion

$y(t) = y_0 \sin(\omega \cdot t + \phi)$ where $\omega = \frac{2\pi}{T}$

T = Period

y_0 = amplitude (distance starting

$y(t) = y_0 \sin\left(\frac{2\pi t}{T}\right)$

- $\omega = \sqrt{\frac{k}{m}}$ — *spring co — *mass

Frequency

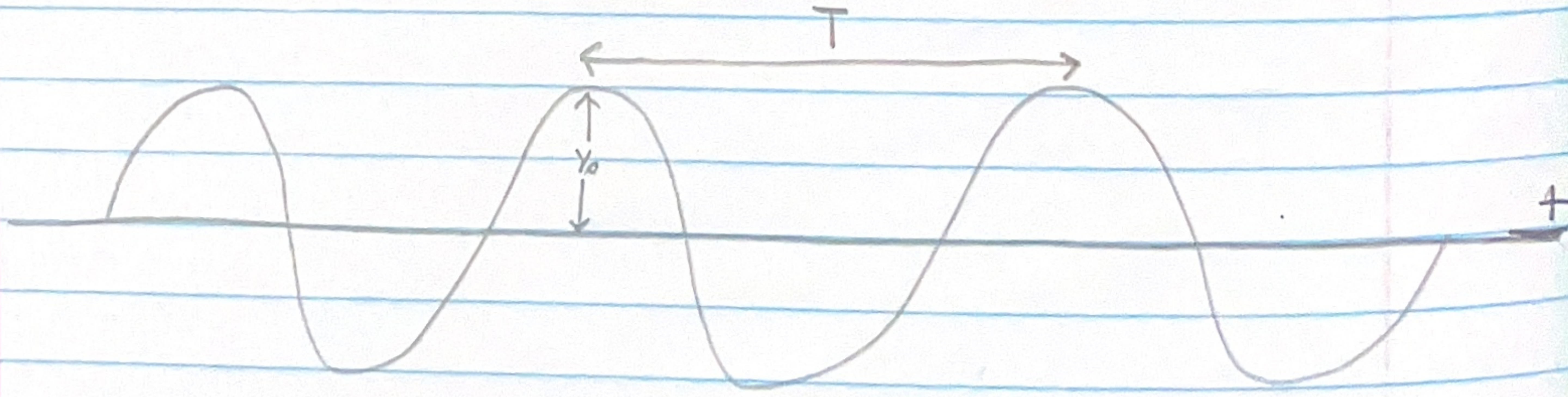
$f = \frac{1}{T}$ $f = \frac{\omega}{2\pi}$

$y(t) = y_0 \sin(\omega \cdot t)$

$y(t) = y_0 \sin\left(\frac{2\pi}{T}\right)$

$y(t) = y_0 \sin(2\pi \cdot f \cdot t)$

- $V_y(t) = y'(t)$
- $a_y(t) = y''(t)$



$V_{MAX} = \left(\sqrt{\frac{k}{m}}\right) y_0$

$V_{MAX} = \omega y_0$